IVIQ Puzzle Competition
MATH Season 2

Problem 1.
Determine the fraction whose numerator and denominator are natural numbers less than 100, which best approximates the number $\sqrt{2}$.

Problem 2.
If all the coefficients in the decomposition of the binomial $(a+b)^n$ are odd, prove that $n = 2^s - 1$, and vice versa.

Problem 3.
We say that a number N has the property $P(k)$ if it can be represented as a product of k consecutive natural numbers greater than 1.

a) Find k for which some number N possesses the properties $P(k)$ and $P(k+2)$.

b) Prove that there is no number that has properties $P(2)$ and $P(4)$.

Problem 4.
Let α and β be real roots of quadratic polynomials with integer coefficients. Prove that $\alpha + \beta$ is a root of a polynomial with integer coefficients of degree no greater than 4.

Problem 5.
Solve the system

$$\cos x_1 = x_2, \cos x_2 = x_3, \ldots, \cos x_n = x_1 .$$

Problem 6.
Can we place an infinite number of circles in a plane such that each line intersects at most two circles?

Problem 7.
If a, b, c, d are the sides of a quadrilateral that is both cyclic and tangential, prove that its area is

$$S = \sqrt{abcd} .$$

Problem 8.
3 vertices emerge from each vertex of the convex polyhedron M. It is known that each of its sides is a polygon around which a circle can be circumscribed. Prove that a sphere can be circumscribed around this polyhedron.

Problem 9.
The centers of three mutually disjoint circular regions lie on one line. If there is a circle that touches all these circular regions, prove that its radius is greater than the radius of at least one of them.

Problem 10.
A finite set of points in the plane is given, so that each has integer coordinates. Is it possible to color some points of that set red, and all the others white, so that for each line L parallel to any of the coordinate axes, the difference between the number of white and red points on L is -1, 0 or 1? Justify the answer.